# Complex Numbers

# Complex numbers

**are numbers** that consist of two parts — a real **number** and an imaginary **number**. **Complex numbers are** the building blocks of more intricate math, such as algebra. They **can** be applied to many aspects of real life, especially in electronics and electromagnetism.

These are all complex numbers: • **1 + I,** • **2 – 6i,** • **–5.2i** (an imaginary number is a complex number with a=0)

• **4** (a real number is a complex number with b=0)

## *Definition of Modulus of a Complex Number:*

- The number of form a+ib where a and b are real numbers i = √-1 is called a
- complex number.It is thus represented as

  z= a+ib or z=x+iy

- Here a or x is called real part of z and is represented as Re(z).

  similarly, b or y is called imaginary part of z and is represented as Im(z)

  The complex number is also represented as z=(a,b) OR z= (x,y)

  Order of these number can not be changed . Thus (a,b) is not equal to (x,y ).

  This means that two complex numbers can be equal only if their real and imaginary parts are equal.

- Modulus of a complex number z = x + iy, denoted by mod(z) or |z| or |x + iy|, is defined as

  square root of ($a^2+b^2$) or square root of ($x^2+y^2$) , it is always taken as positive root.

- Modulus of complex number plays important role in theory of complex variable.

- The **complex conjugate** of a complex number is the number with an equal real part and an imaginary part equal in magnitude but opposite in sign

  For example, (if *a* and *b* are real, then) the complex conjugate of a + b I,is a-bi

# Properties of complex numbers.

- When a, b are real numbers and $a + ib = 0$ then

  $a = 0, b = 0$
- When a, b, c and d are real numbers and $a + ib = c + id$ then $a = c$ and $b = d$.
- The sum of two conjugate complex numbers is real.
- The product of two conjugate complex numbers is real.
- When the sum of two complex numbers is real and the product of two complex numbers is also real then the complex numbers are conjugate to each other.

* ALGEBRA OF COMPLEX NUMBERS
* Fundamental laws of algebra are obeyed by Complex numbers.
* They obey most of the properties of real numbers.

Take two complex numbers, $z_1 = x_1 + y_1\, i$ and $z_2 = x_2 + y_2\, i$

**Addition:**

We obtain:

$z_1 + z_2 = ( x_1 + y_1 i ) + ( x_2 + y_2\, i ) = (x_1+x_2) + ( y_1+y_2)\, i$

**Subtraction:**

$z_1 - z_2 = ( x_1 + y_1\, i ) - ( x_2 + y_2\, i ) = ( x_1 - x_2 ) + ( y_1 - y_2 )\, i$

**Multiplication:**

$z1\, z_2 = ( x_1 + y_1\, i ) ( x_2 + y_2\, i ) = ( x_1\, x_2 - y_1 y_2) + ( y_1 x_2 + y_2\, x_1 )\, i$

**Division:**

$z1/z2 = ( x_1 + y_1\, i ) / ( x_2 + y_2\, i ) = ( x_1 + y_1\, i ) ( x_2 - y_2\, i ) / ( x_2 + y_2\, i ) ( x_2 - y_2)\, i$

$=(x_1 x_2+y_1 y_2)/x_2^2+y_2^2 +(y_1 x_2-x_1 y_2)i/ x_2^2+y_2^2$

# SOME OTHER PROPERTIES OF COMPLEX NUMBER

* 1.Commutativity of addition:

  $z_1+z_2=z_2+z_1$

  2. Commutativity of Multiplication:

  $z_1z_2=z_2z_1$

  3. Associativity of addition:

  $(z_1+z_2)+z_3=z_1+(z_2+z_3)$

  4.Associativity of Multiplication:

  $(z_1z_2)z_3=z_1(z_2z_3)$

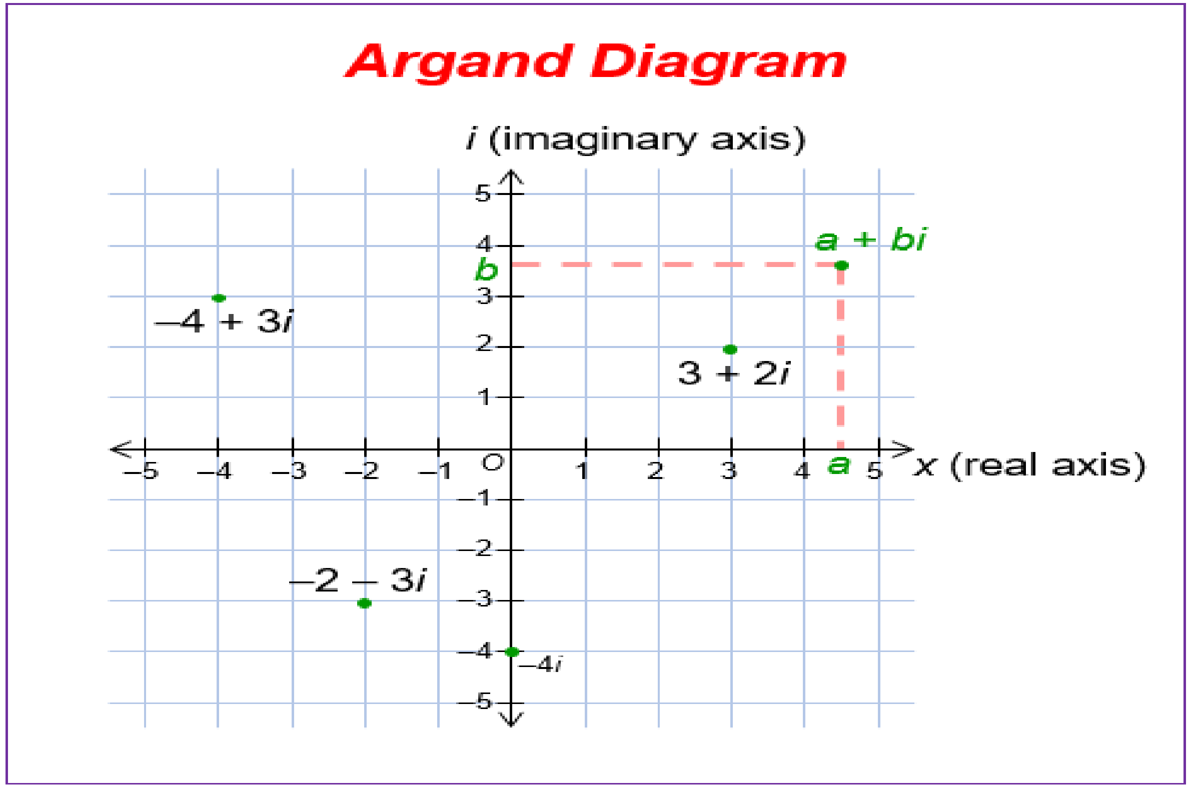
  5.Distributivity of multiplication:

  $(z_1+z_2)z_3=z_1z_3+z_2z_3$

  6. Equality:

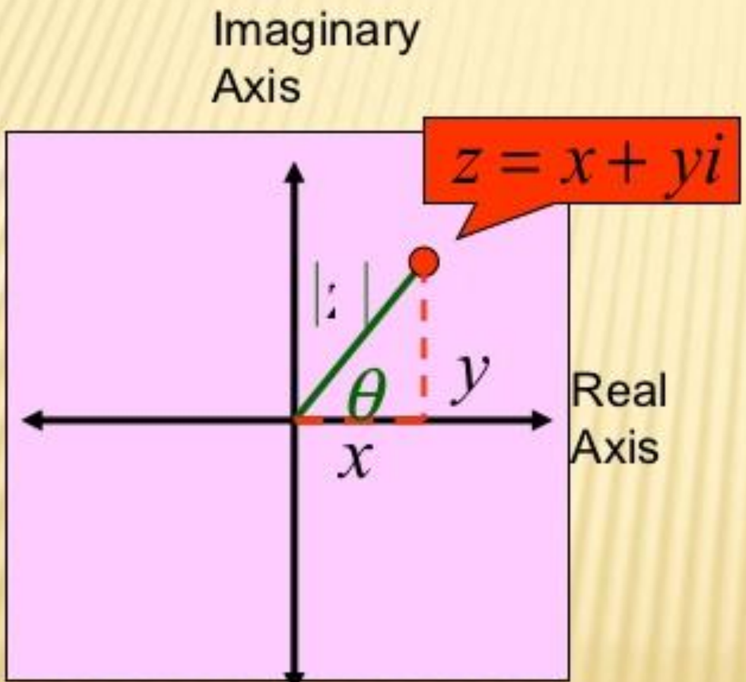
  Two complex numbers are equal only when their real parts and imaginary parts are equal.

Argand diagram is the graphical representation of complex number



# Modulus and Argument of Complex Numbers

Let a complex number be Z such that : $z = x + yi$




$$|z| = \sqrt{x^2 + y^2}$$

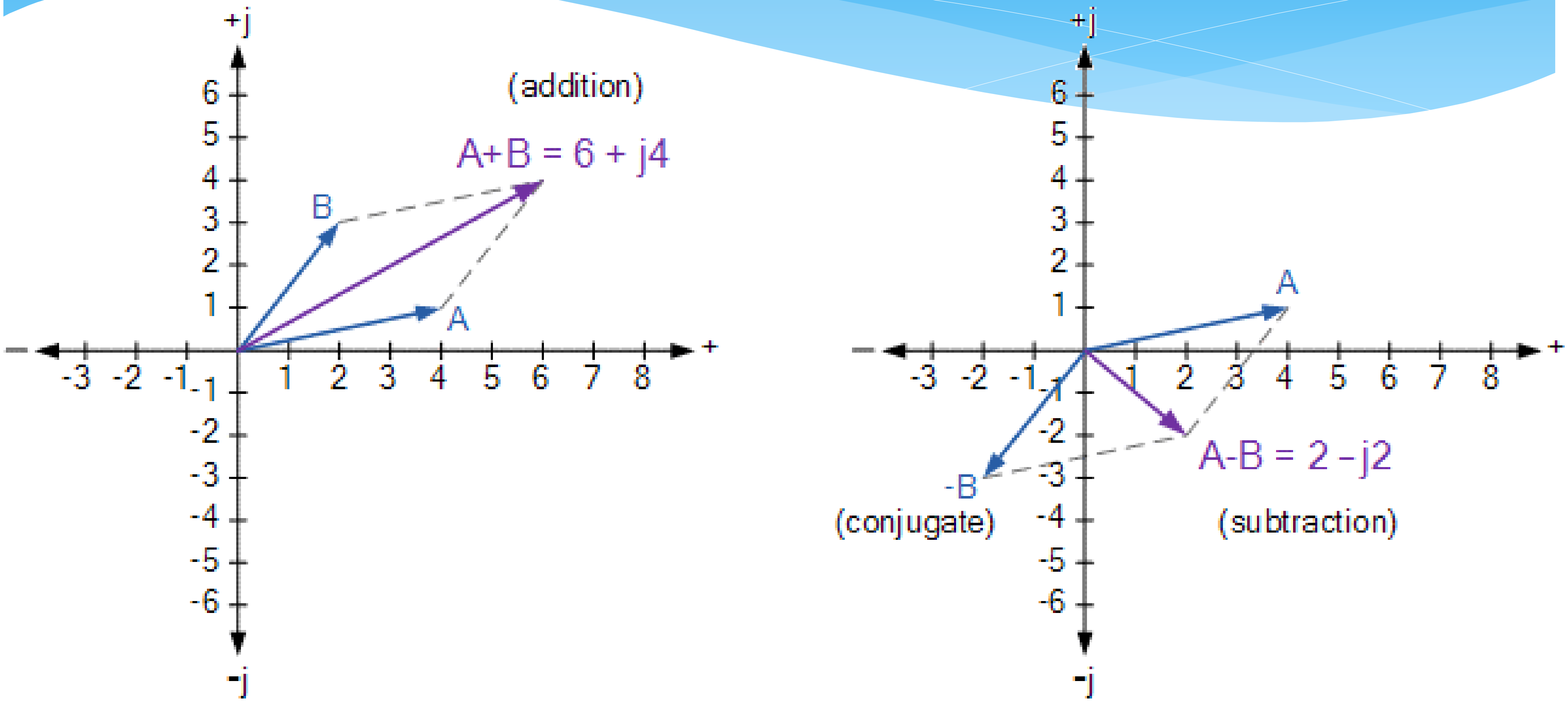
The magnitude or modulus of $z$ denoted by $|z|$ is the distance from the origin to the point $(x, y)$.

The angle formed from the real axis and a line from the origin to $(x, y)$ is called the argument of z, with requirement that $0 \leq \theta < 2\pi$.

modified for quadrant and so that it is between 0 and $2\pi$

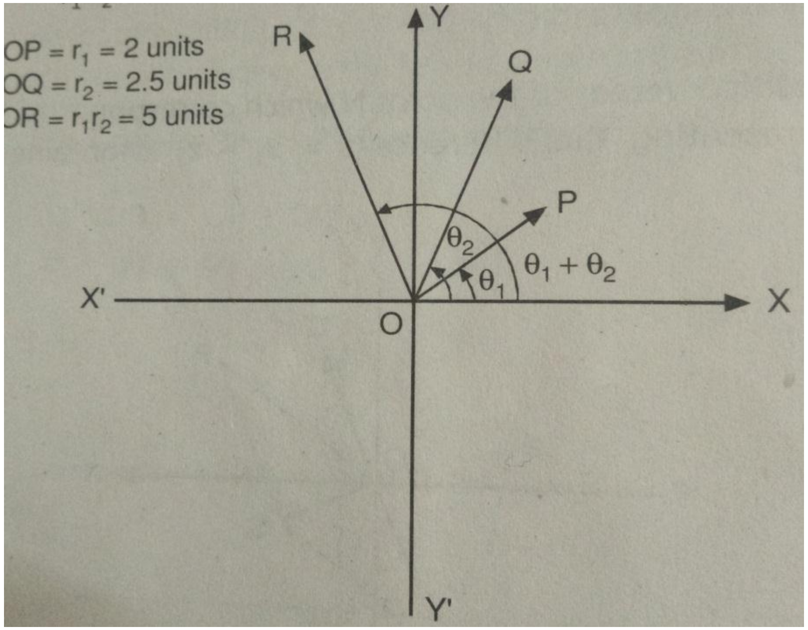
$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

# Addition and Subtraction of complex numbers using Argand Diagram.

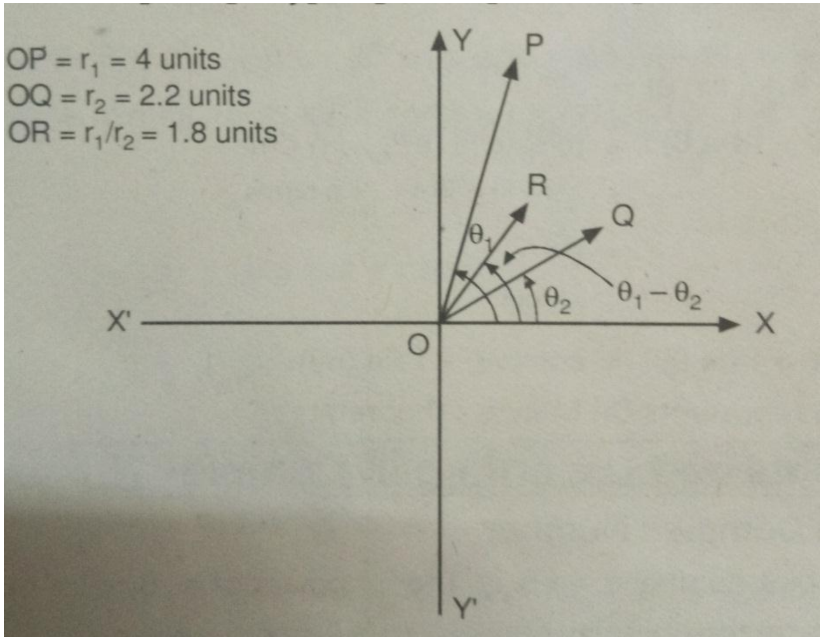


# Multiplication and Division of complex numbers using Argand Diagram.

## Multiplication




## Division



# De Moivre's Theorem

- Statement-

$(\cos\theta + i\sin\theta)^n = \cos(n\theta) + i\sin(n\theta)$

Where n is a power

This theorem is useful to find the power and root of complex number.

**Statement :**

$$(\cos\theta + i\sin\theta)^n = \cos(n\theta) + i\sin(n\theta)$$

where n is a power.

Now $(\cos\theta + i\sin\theta) = e^{i\theta}$

$\therefore$ $(\cos\theta + i\sin\theta)^n = (e^{i\theta})(e^{i\theta})(e^{i\theta}) \ldots$ n times

$= e^{i\theta + i\theta + i\theta + \ldots + n \text{ terms}}$

$= e^{in\theta}$

$= e^{i(n\theta)}$

$\therefore$ $(\cos\theta + i\sin\theta)^n = \cos(n\theta) + i\sin(n\theta)$

Above equation represents De Moivre's theorem.

# FORMS OF COMPLEX NUMBER

## RECTANGULAR FORM

Z=x+iy is complex number where x and y are rectangular co-ordinates

## POLAR FORM

$Z= r(\cos\theta+i\sin\theta)$

Here $x= r\cos\theta$ , $y=r\sin\theta$

and $x^2+y^2=r^2$

## EXPONENTIAL FORM

$Z= re^{i\theta}$