*Graphs:-

*Defn:- If f is a function of two variables with domain D, then the graph of f is the set of all points (x, y, z) in R^3 such that $z = f(x, y)$ and $(x, y) \in D$.

*Note:- As the graph of a function of one variable is a curve "C" with the equation $y = f(x)$ and so the graph of a function of two variables is a surface "S" with the equation $z = f(x, y)$.

ex ①: Sketch the graph of the function $f(x, y) = 6 - 3x - 2y$.

Soln:- The graph of f has the equation $z = 6 - 3x - 2y \Rightarrow 3x + 2y + z = 6$ which is the equation of plane. —①

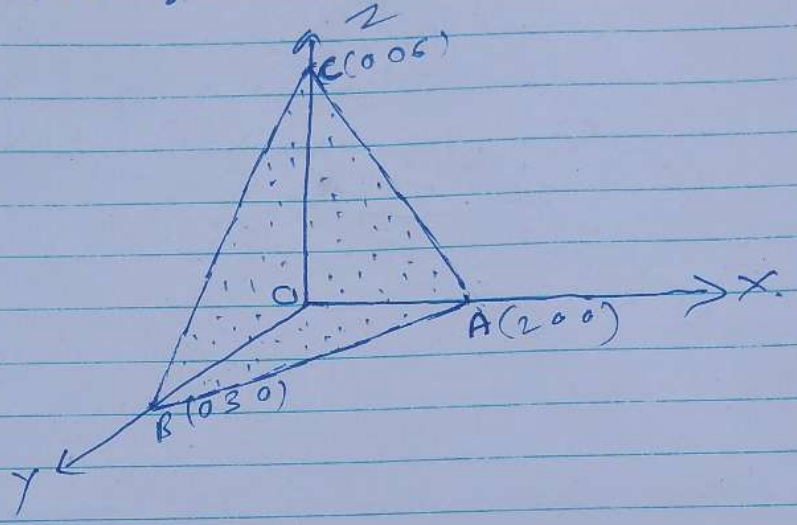
To find the graph of this plane we have to find the intercepts on these co-ordinate axes.

Put $y = z = 0$ in eq.n ① we have $x = 2$ is the x-intercept. i.e. plane ① cuts X-axis at the pt. $(2\ 0\ 0)$

Also put $x = z = 0$ in eq.n ① we have $y = 3$ is the Y-intercept. i.e. plane ① cuts Y-axis at the pt. $(0\ 3\ 0)$

And put $x = y = 0$ in eq.n ① we have $z = 6$

is z-intercept. i.e. plane ① cuts z-axis at the pt. (0 0 6).
∴ The graph of this plane is



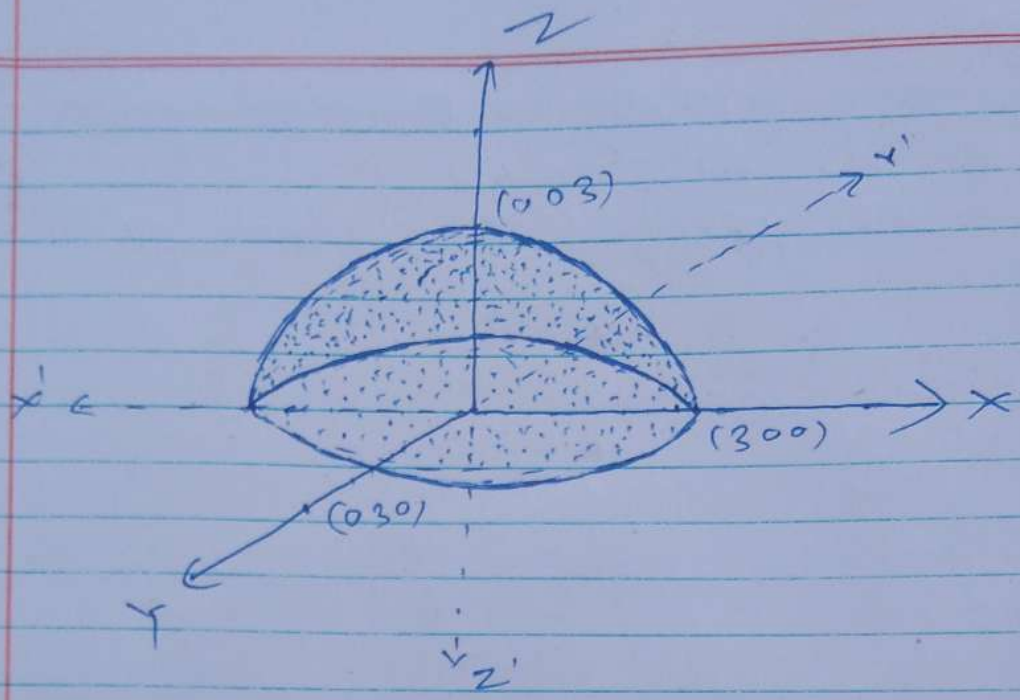
* Note :- Above function is the special case of the function $f(x,y) = ax+by+c$ i.e. $z = ax+by+c$ i.e. $ax+by-z+c=0$ which is the equation of plane.

ex ②: Sketch the graph of the function $f(x,y) = \sqrt{9-x^2-y^2}$.

Soln :- Given function is $f(x,y) = \sqrt{9-x^2-y^2}$

$\Rightarrow z = \sqrt{9-x^2-y^2} \Rightarrow z^2 = 9-x^2-y^2 \Rightarrow x^2+y^2+z^2 = 9$

which is the equation of standard sphere with centre at O(0 0 0) and radius 3.
As the value of z is positive square root, we have $z \geq 0$ and so the graph of function f is just the top half of the sphere $x^2+y^2+z^2=9$



ex (3) Sketch the graph of the function
$f(x,y) = 10 - 5x - 5y$

Soln: $f(x,y) = 10 - 5x - 5y \Rightarrow z = 10 - 5x - 5y$

$\Rightarrow 5x + 5y + z = 10$ ——— (1)
is the eqn of plane

Put $y = z = 0 \Rightarrow 5x = 10 \Rightarrow x = 5/2$

∴ The pt. of intercept is $(5/2\ 0\ 0)$

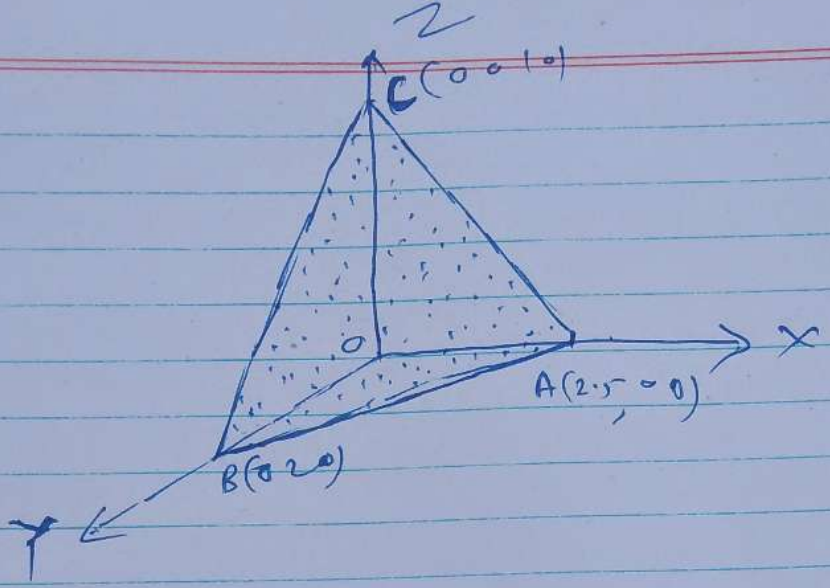
Put $x = z = 0 \Rightarrow 5y = 10 \Rightarrow y = 2$

The pt. of intercept is $(0\ 2\ 0)$

Put $x = y = 0 \Rightarrow z = 10$

∴ The pt. of intercept is $(0\ 0\ 10)$

∴ The equation of plane (1) is the plane in intercept form which cuts x-axis at $A(\frac{5}{2}\ 0\ 0)$, y-axis at $B(0\ 2\ 0)$ & z-axis at $C(0\ 0\ 10)$


Z
C(0 0 10)
O
X
A(2.5, 0 0)
B(0 2 0)
Y