1.6 Maximum Power Transfer Theorem

It states that maximum power will be delivered to a load resistance R_L, if the load resistance (R_L) connected to the network is equal to the internal resistance (R_i) of the network delivering the power.

Consider a network consisting of voltage sources and some resistances. A load resistance R_L is connected across the terminals A and B of a network (Refer Fig. 1.40). Using Thevenin's theorem, this network (inside the box) can be replaced by an equivalent circuit consisting of V_{Th} (or V_{OC}) in series with R_{Th} (or R_{in}) as shown in Fig. 1.41.

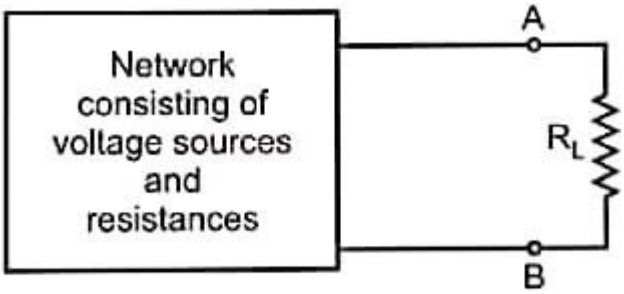


Fig. 1.40 : Black box connected to the load

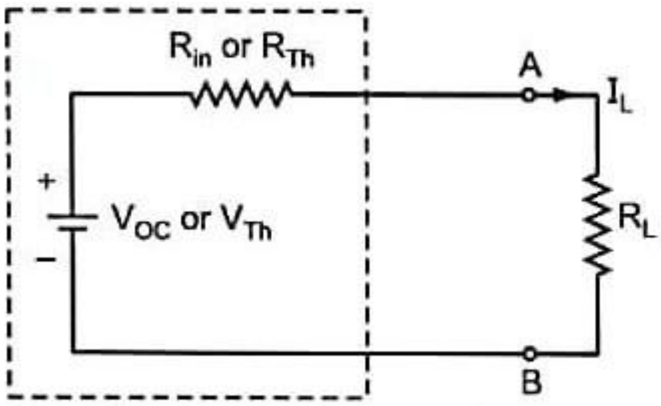


Fig. 1.41 : Equivalent circuit

In Fig. 1.41, R_{in} is the internal resistance of the network.

According to this theorem, R_L will extract maximum power from the network when $R_L = R_{in}$. To prove the theorem, consider Thevenin's equivalent of the given circuit as shown in Fig. 1.41.

Current passing through R_L is $$I_L = \frac{V_{OC}}{R_{in} + R_L}$$

Power delivered to the load is $$P_L = I_L^2 R_L = \frac{V_{OC}^2}{(R_L + R_{in})^2} R_L \quad \text{... (1.20)}$$

From the above equation it is clear that power delivered to a load depends on load resistance R_L.

If load resistance is varied, power transferred to the load changes. At a particular value of R_L, maximum power is transferred from a network to the load. For power (P_L) to be maximum,

$$\frac{dP_L}{dR_L} = 0$$

Differentiating equation (1.20) w.r.t. to R_L, we get

$$\frac{dP_L}{dR_L} = \frac{d}{dR_L}\left[\frac{V_{OC}^2 R_L}{(R_L + R_{in})^2}\right]$$

$$= V_{OC}^2 \frac{d}{dR_L}[R_L (R_L + R_{in})^{-2}]$$

$$= V_{OC}^2 [1 \times (R_L + R_{in})^{-2} + R_L (-2) (R_L + R_{in})^{-3}]$$

$$= V_{OC}^2 [(R_L + R_{in})^{-2} - 2R_L (R_L + R_{in})^{-3}]$$

$$= V_{OC}^2 (R_L + R_{in})^{-3} [(R_L + R_{in}) - 2R_L]$$

$$\frac{dP_L}{dR_L} = \frac{R_{in} - R_L}{(R_L + R_{in})^3} V_{OC}^2$$

But for maximum power transfer, we have $\frac{dP_L}{dR_L} = 0$

$$\therefore \qquad \frac{(R_{in} - R_L) V_{OC}^2}{(R_L + R_{in})^3} = 0$$

The L.H.S. of above equation becomes zero, if $R_L = R_{in}$

Thus R_L will extract maximum power from the network when R_L becomes equal to R_{in}. Hence the proof of the theorem.

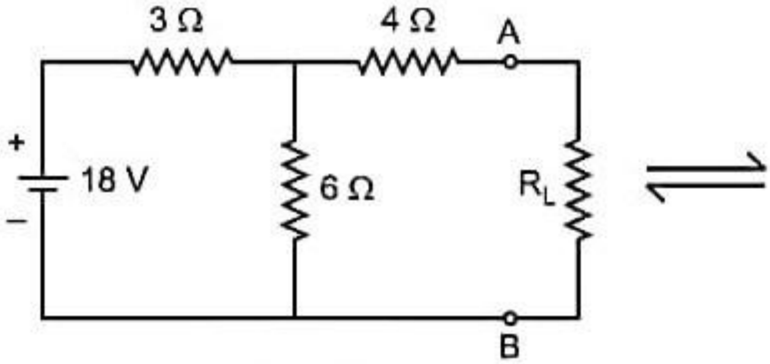
The expression for maximum power can be obtained by using $R_{in} = R_L$ in equation (1.22).

Thus, $$(P_L)_{max} = \frac{V_{OC}^2 R_L}{4R_L^2}$$

$$(P_L)_{max} = \frac{V_{OC}^2}{4R_L}$$

This is the expression for maximum power which can be delivered to the load resistance R_L.

Problem 6 : *For the given circuit, calculate the value of R_L for which the power dissipated in it would be maximum. Also find the value of this maximum power.*



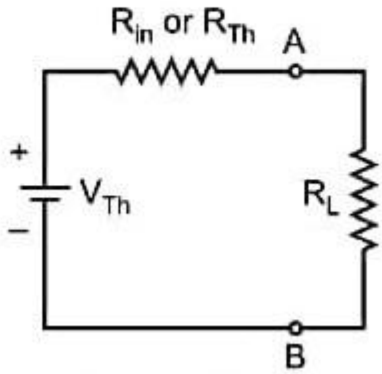


Fig. 1.62 **Fig. 1.63**

Solution : Using Thevenin's theorem, given network can be replaced by an equivalent circuit as shown in Fig. 1.63.

Let us find R_{in} and V_{Th} or V_{OC}.

To find V_{OC}, remove load resistor R_L from terminals A and B. Fig. 1.64 shows the circuit with terminals A and B open.

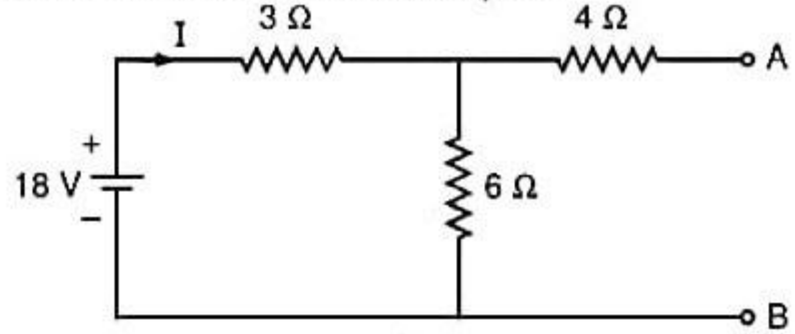


Fig. 1.64

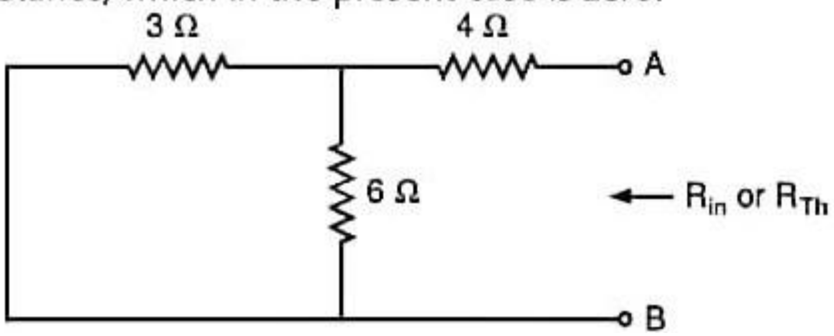
Current flowing through 6 Ω resistor is $I = \dfrac{18}{3+6} = 2$ A

The voltage drop across 6 Ω resistance is

$$V_{OC} = V_{Th} = 6 \times 2 = 12 \text{ V}.$$

Thus, we get $V_{Th} = 12$ V.

To find R_{in} or R_{Th}, remove the 18 volts battery leaving behind its internal resistance, which in the present case is zero.



From Fig. 1.65, $R_{Th} = (3 \parallel 6) + 4\ \Omega$

$$= \frac{3 \times 6}{3 + 6} + 4\ \Omega = 2 + 4 = 6\ \Omega$$

$$\therefore \quad R_{in} = R_{Th} = 6\ \Omega$$

According to maximum power transfer theorem, the power drawn by R_L from the network will be maximum when $R_L = R_{in}$ (or $R_L = R_{Th}$).

Thus, maximum power will be drawn by R_L when $R_L = 6\ \Omega$.

Let us find the value of maximum power.

The current flowing through R_L (Fig. 1.63) is

$$I = \frac{V_{Th}}{R_L + R_{in}}$$

But $R_L = R_{in}$

$$\therefore \quad I = \frac{V_{Th}}{2\ R_L}$$

Maximum power drawn by

$$R_L = I^2\ R_L = \frac{V_{Th}^2}{4\ R_L^2} \cdot R_L = \frac{V_{Th}^2}{4\ R_L} = \frac{12 \times 12}{4 \times 6} = 6\ \text{W}$$

For any other value of R_L, power drawn will be less than 6 W.