1.5 Superposition Theorem

If a network contains more than one energy sources that are not in series or parallel, then analysis of such network on the basis of Kirchhoff's laws becomes very long and laborious. In such a case, the analysis becomes easier if we use the Superposition theorem.

Statement of the theorem :

In a linear bilateral network, if there are number of voltage or current sources acting simultaneously, then the current through or voltage across any element is equal to the algebraic sum of the currents or voltages produced independently by each source.

Problem 2 : *Using superposition theorem, find the current passing through 3Ω resistance of the following circuit.*

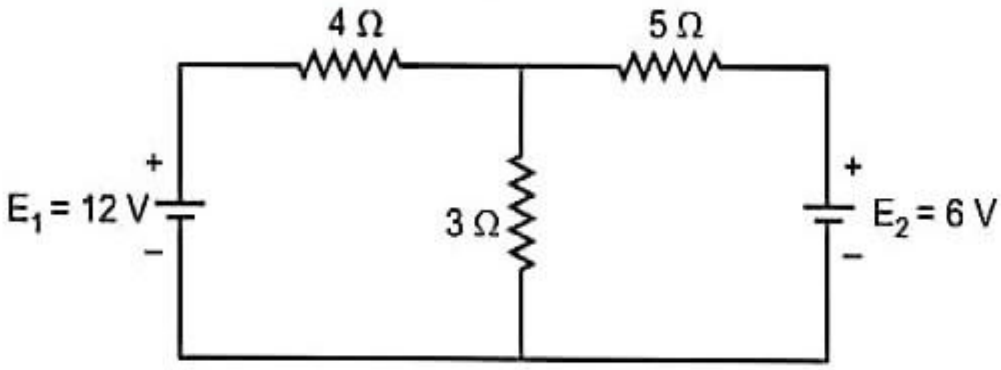

Fig. 1.37

Solution : Step I : To find the effect of 12 volts d.c. source, short circuit the 6 volts source as shown in Fig. 1.38.

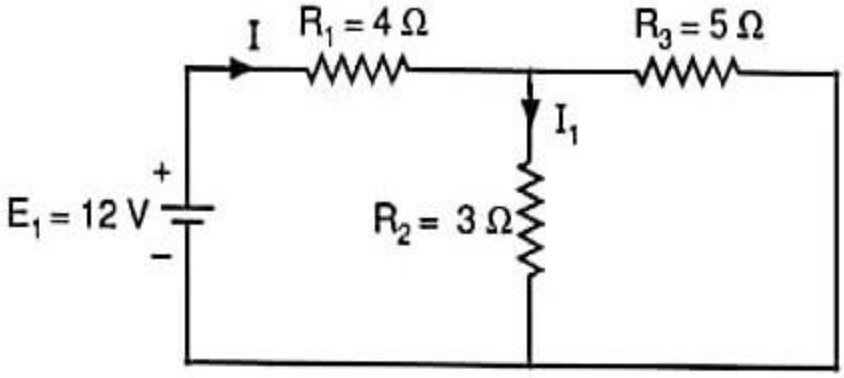

Fig. 1.38

The equivalent resistance of the circuit is

$$= 4 + (5 \parallel 3)$$

$$= 4 + \frac{3 \times 5}{3 + 5} = 4 + \frac{15}{8} = 5.87\ \Omega$$

The current drawn from 12 volts d.c. source is

$$I = \frac{12}{5.87} = 2.04\ \text{A}$$

Current through 3 Ω resistor due to 12 volts source is

$$I_1 = I \times \frac{R_3}{R_3 + R_2} = I \times \frac{5}{5 + 3}$$

$$= 2.04 \times \frac{5}{8} = 1.27\ \text{A}$$

Step II : To find the effect of 6 volts d.c. source, short circuit the 12 volts source (Refer Fig. 1.39).

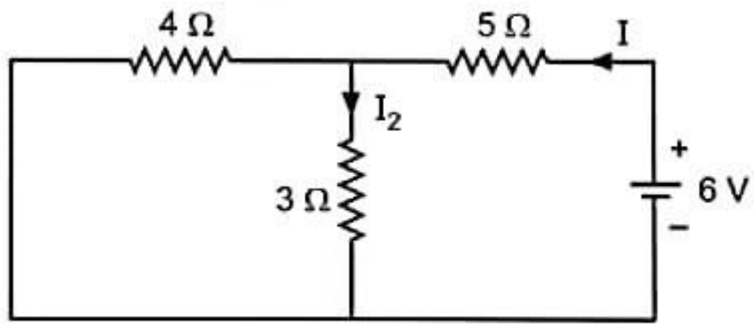

Fig. 1.39

The equivalent resistance of the circuit is

$$= 5 + (4 \parallel 3) = 5 + \frac{4 \times 3}{4 + 3} = 5 + \frac{12}{7} = 6.71\ \Omega$$

The current drawn from 6 volts d.c. source is

$$I = \frac{6\ \text{V}}{6.71} = 0.89\ \text{A}$$

Current through 3 Ω resistor due to 6 V source is

$$I_2 = I \times \frac{R_1}{R_1 + R_2} = I \times \frac{4}{4 + 3}$$

$$= 0.89 \times \frac{4}{7} = 0.508\ \text{A}$$

Total current through 3 Ω resistance = $I_1 + I_2 = 1.778$ A

Problem 5 : *Using superposition theorem, determine the current through 4 Ω resistor in the circuit shown in Fig. 1.59.*

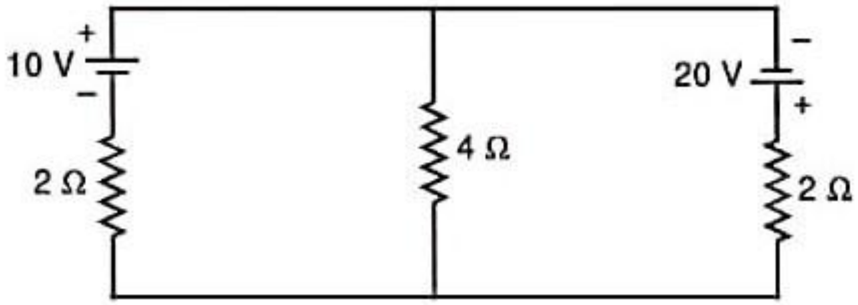

Fig. 1.59

Solution : Step I : To find the effect of 10 volts source, a battery of 20 volts is removed leaving behind its internal resistance, which in the present case is zero.

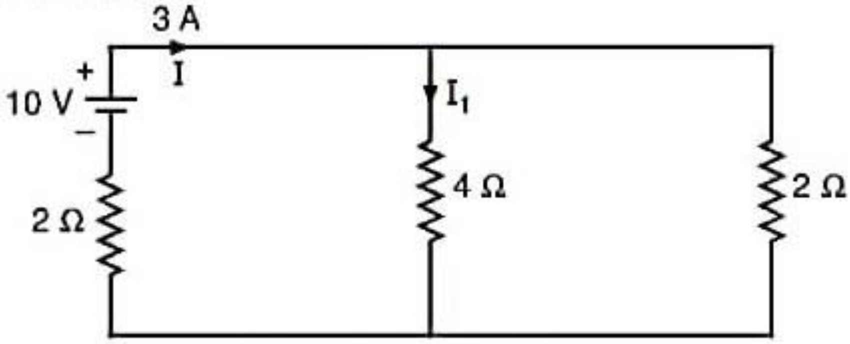

Fig. 1.60

From Fig. 1.60, total resistance of the circuit is

$$R = 2 + (4 \parallel 2) = 2 + \frac{4 \times 2}{4 + 2} = 2 + \frac{4}{3} = \frac{10}{3}\ \Omega$$

Total circuit current is $I = \frac{10}{10/3} = 3$ A

Current flowing through 4 Ω resistor is

$$I_1 = I \times \frac{2}{4 + 2} = 3 \times \frac{2}{6} = 1\ A$$

Step II : To find the effect of 20 volts battery, a battery of 10 volts is removed, leaving behind its internal resistance, which in the present case is zero.

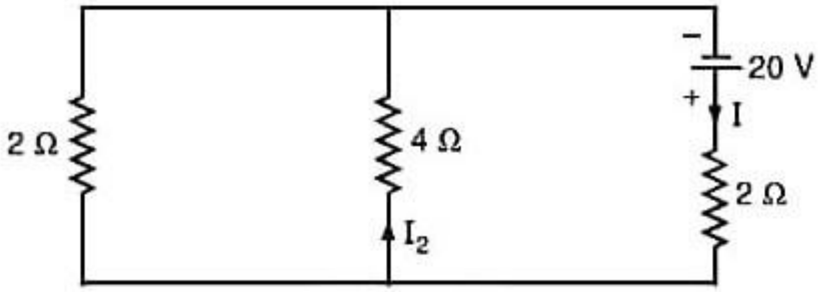

Fig. 1.61

From Fig. 1.61, total resistance of the circuit is

$$R = (2 \parallel 4) + 2 = \frac{2 \times 4}{2 + 4} + 2 = \frac{10}{3}\,\Omega$$

Total circuit current is $I = \dfrac{20}{10/3} = 6$ A

Current flowing through 4 Ω resistor is

$$I_2 = I \times \frac{2}{4 + 2} = 6 \times \frac{2}{6} = 2 \text{ A}$$

Total current flowing through 4 Ω resistor is

$$I_2 - I_1 = 2 \text{ amp.} - 1 \text{ amp.} = 1 \text{ A}$$
