In statistics, the **Arithmetic Mean (AM)** or called average is the ratio of the sum of all observations to the total number of observations. The arithmetic mean can also inform or model concepts outside of statistics. In a physical sense, the arithmetic mean can be thought of as a centre of gravity. From the mean of a data set, we can think of the average distance the data points are from the mean as standard deviation. The square of standard deviation (i.e. variance) is analogous to the moment of inertia in the physical model.

Say, for example, you wanted to know the weather in Shimla. On the internet, you would find the temperatures for a lot of days, data of the temperature in the past and the data of the temperature in the present and also the predictions of the temperature in the future. Wouldn't all this be extremely confusing? Instead of this long list of data, mathematicians decided to use representative values that could take into consideration a wide range of data. Instead of weather for every particular day, we use terms such as average (arithmetic mean), median and mode to describe weather over a month or so. There are several types of representative values that are used by Mathematicians in data handling, namely;

- Arithmetic Mean (Average)
- Range
- Median
- Mode

Out of the four above, *mean, median* and *mode* are types of *average*

Read: Mean, Median and Mode

What is Arithmetic Mean?

Arithmetic mean represents a number that is obtained by dividing the sum of the elements of a set by the number of values in the set. So you can use the layman term Average, or be a little bit fancier and use the word "Arithmetic mean" your call, take your pick -they both mean the same. The arithmetic mean may be either

- Simple Arithmetic Mean
- Weighted Arithmetic Mean

Arithmetic Mean Formula

If any data set consisting of the values b_1, b_2, b_3, …., b_n then the arithmetic mean B is defined as:

B = (Sum of all observations)/ (Total number of observation)

= 1/n ∑ni=1bi=b1+b2+b3+….+bnn∑i=1nbi=b1+b2+b3+….+bnn
If these n observations have corresponding frequencies, the arithmetic mean is computed using the formula

x = x1f1+x2f2+……+xnfnNx1f1+x2f2+……+xnfnN and
using Sigma notation = ∑ni=1xifiN∑i=1nxifiN
where N = f_1+ f_2 + ………+ f_n.

The above formula can also be used to find the weighted arithmetic mean by taking f_1, f_2,...., f_n as the weights of x_1, x_2,....., x_n.

When the frequencies divided by N are replaced by probabilities p_1, p_2, ...,p_n we get the formula for the expected value of a discrete random variable.

$X = x_1p_1 + x_2p_2 + ... + x_np_n$. or

using Sigma notation = ∑ni=1xipi∑i=1nxipi

Properties of Arithmetic Mean

Some important properties of the arithmetic mean are as follows:

- The sum of deviations of the items from their arithmetic mean is always zero, i.e. $\sum(x - X) = 0$.
- The sum of the squared deviations of the items from Arithmetic Mean (A.M) is minimum, which is less than the sum of the squared deviations of the items from any other values.
- If each item in the arithmetic series is substituted by the mean, then the sum of these replacements will be equal to the sum of the specific items.

Arithmetic Mean of Ungrouped Data

For ungrouped data, we can easily find the arithmetic mean by adding all the given values in a data set and dividing it by a number of values.

Mean, $\bar{x}$ = Sum of all values/Number of values

Example: Find the arithmetic mean of 4, 8, 12, 16, 20.

Solution: Given, 4, 8, 12, 16, 20 is the set of values.

Sum of values = 4+ 8+12+16+20 = 60

Number of values = 5

Mean = 60/5 = 12

Arithmetic Mean of Ungroued Data

If x_1, x_2, x_3 ,......,x_n be the observations with the frequencies f_1, f_2, f_3 ,......,f_n, then the arithmetic mean is given by:

$\bar{x} = (x_1f_1+x_2f_2+......+x_nf_n) / \sum f_i$

where $\sum f_i$ is the summation of all the frequencies.

Let us understand the arithmetic mean of ungrouped data with the help of an example.

Example: Find the mean of given distribution:

x	10	20	30	40	50
f	3	2	4	5	1

Solution: Let us find the value of x_if_i and $\sum f_i$, for all the values of x and f respectively.

xi	fi	xifi
10	3	10 x 3 = 30
20	2	20 x 2 = 40
30	4	30 x 4 = 120
40	5	40 x 5 = 200
50	1	50 x 1 = 50
Total	∑fi = 15	∑xifi = 440

Hence, the required mean is:

$\bar{x}$ = 440/15 = 29.33

Merits of Arithmetic Mean

- The arithmetic mean is simple to understand and easy to calculate.
- It is influenced by the value of every item in the series.
- A.M is rigidly defined.
- It has the capability of further algebraic treatment.
- It is a measured value and not based on the position in the series.

Demerits of Arithmetic Mean

- It is changed by extreme items such as very small and very large items.
- It can rarely be identified by inspection.
- In some cases, A.M. does not represent the original item. For example, the average number of patients admitted to a hospital are 10.7 per day.
- The arithmetic mean is not suitable in extremely asymmetrical distributions.

Representative Values of Data

We see the use of representative value quite regularly in our daily life. When you ask about the mileage of the car, you are asking for the representative value of the amount of distance travelled to the amount of fuel consumed. This doesn't mean that the temperature in Shimla in constantly the representative value but that overall, it amounts to the average value. Average here represents a number that expresses a central or typical value in a set of data, calculated by the sum of values divided by the number of values.

Arithmetic Mean Example

Arithmetic means utilizes two basic mathematical operations, addition and division to find a central value for a set of values.

Example: If you wanted to find the arithmetic means of the runs scored by Virat Kohli in the last few innings, all you would have to do is sum up his runs to obtain sum total and then divide it by the number of innings. For example;

Innings	1	2	3	4	5	6	7	8	9	10
Runs	50	59	90	8	106	117	59	91	7	74

The arithmetic mean of Virat Kohli's batting scores also called his Batting Average is;

Sum of runs scored/Number of innings = 661/10

The arithmetic mean of his scores in the last 10 innings is 66.1. If we add another score to this sum, say his 11th innings, the arithmetic mean will proportionally change. If the runs scored in 11th innings are 70, the new average becomes;

$$\frac{661 + 70}{10 + 1} = \frac{731}{11} \quad 66.45$$

The average is a pretty neat tool, but it comes with its set of problems. Sometimes it doesn't represent the situation accurately enough. I'll show you what I mean. Let's take the results of a class test, for example. Say there are 10 students in the class and they recently gave a test out of 100 marks. There are two scenarios here.

First: 50, 53, 50, 51, 48, 93, 90, 92, 91, 90

Second: 71, 72, 70, 75, 73, 74, 75, 70, 74, 72

Why don't you calculate the Arithmetic mean of both the sets above? You will find that both the sets have a huge difference in the value even though they have similar arithmetic mean. In this respect, completely relying on arithmetic mean can be occasionally misleading. At least from the point of view of students scoring 50's/ 100, the second scenario is quite different. The same applies to the students with 90, in the case of these students in the second set, the marks are reduced. So for both the classes, the results mean something different, but the average for both classes are the same. In the first class, the students are performing very varied, some very well and some not so well whereas in the other class the performance is kind of uniform. Therefore we need an extra representative value to help reduce this ambiguity.

Arithmetic Range

Range, as the word suggests, represents the difference between the largest and the smallest value of data. This helps us determine the range over which the data is spread—taking the previous example into consideration once again. There are 10 students in the class, and they recently gave a test out of 100 marks. There are two scenarios here.

First: 50, 53, 50, 51, 48, 93, 90, 92, 91, 90

Second: 71, 72, 70, 75, 73, 74, 75, 70, 74, 72

The range in the first scenario is represented by the difference between the largest value, 93 and the smallest value, 48.

Range in First set = 93 – 48 = 45

Whereas in the second scenario, the range is represented by the difference between the highest value, 75 and the smallest value, 70.

Range in the second set = 75 – 70 = 5

The difference in the value of range between the two scenarios enables us to estimate the range over which the values are spread, the larger the range, the larger apart the values are spread. This gives us the extra information which is not getting through on average.

Median, in statistics, is the middle value of the given list of data, when arranged in an order. The arrangement of data or observations can be done either in ascending order or descending order.

Example: The median of 2,3,4 is 3.

In Maths, the median is also a type of average, which is used to find the center value. Therefore, it is also called measure of central tendency.

Apart from the median, the other two central tendencies are mean and mode. Mean is the ratio of sum of all observations and total number of observations. Mode is the value in the given data-set, repeated most of the time.

Learn more:

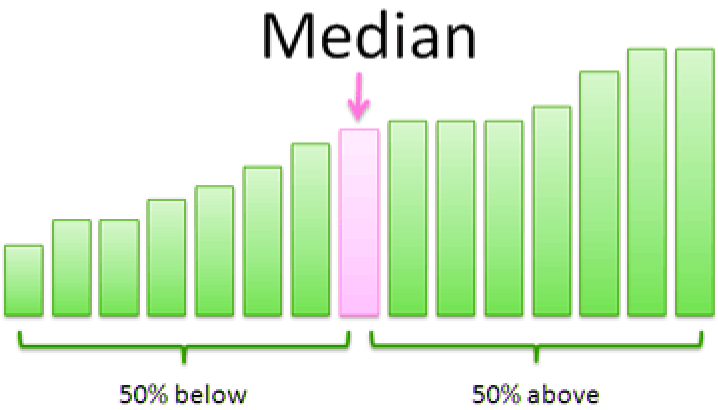
- Mean
- Mode

In geometry, a median is also defined as the center point of a polygon. For example, the median of a triangle is the line segment joining the vertex of triangle to the center of the opposite sides. Therefore, a median bisects the sides of triangle.

Median in Statistics

The median of a set of data is the middlemost number or center value in the set. The median is also the number that is halfway into the set.

To find the median, the data should be arranged, first, in order of least to greatest or greatest to the least value. A median is a number that is separated by the higher half of a data sample, a population or a probability distribution, from the lower half. The median is different for different types of distribution.



For example, the median of 3, 3, 5, 9, 11 is 5. If there is an even number of observations, then there is no single middle value; the median is then usually defined to be the mean of the two middle values: so the median of 3, 5, 7, 9 is (5+7)/2 = 6.

Also, read:

- Mean, Median and Mode Formula
- Difference Between Mean, Median and Mode
- Relation Between Mean, Median and Mode

Median Formula

The formula to calculate the median of the finite number of data set is given here. Median formula is different for even and odd numbers of observations. Therefore, it is necessary to recognise first if we have odd number of values or even number of values in a given data set.

The formula to calculate the median of the data set is given as follow.

Odd Number of Observations

If the total number of observation given is odd, then the formula to calculate the median is:

$$\textbf{Median} = \{(n+1)/2\}^{th}\textbf{term}$$

where n is the number of observations

Even Number of Observations

If the total number of observation is even, then the median formula is:

$$\text{Median} = [(n/2)^{th}\text{ term} + \{(n/2)+1\}^{th}]/2$$

where n is the number of observations

How to Calculate the Median?

To find the median, place all the numbers in the ascending order and find the middle.

Example 1:

Find the Median of 14, 63 and 55

solution:

Put them in ascending order: 14, 55, 63

The middle number is 55, so the median is 55.

Example 2:

Find the median of the following:

4, 17, 77, 25, 22, 23, 92, 82, 40, 24, 14, 12, 67, 23, 29

Solution:

When we put those numbers in the order we have:

4, 12, 14, 17, 22, 23, 23, 24, 25, 29, 40, 67, 77, 82, 92,

There are fifteen numbers. Our middle is the eighth number:

The median value of this set of numbers is 24.

Example 3:

Rahul's family drove through 7 states on summer vacation. The prices of Gasoline differ from state to state. Calculate the median of gasoline cost.

1.79, 1.61, 2.09, 1.84, 1.96, 2.11, 1.75

Solution:

By organizing the data from smallest to greatest, we get:

1.61, 1.75, 1.79, **1.84** , 1.96, 2.09, 2.11

Hence, the median of the gasoline cost is 1.84. There are three states with greater gasoline costs and 3 with smaller prices.

Mean Median Mode

Let us see an example here to find mean, median and mode of the observations.

For example, 2,6,9,12,12 is the given set of data

Thus,

Median = Middle Value = 9

Mean = Sum of observations/Number of observations = (2+7+9+12+12)/5 = 41/5 = 8.2

Mode = Value repeated most number of times = 12

For more Maths-related articles, register with BYJU'S – The Learning App and download the app to learn with ease.

Frequently Asked Questions – FAQs

What is the Median? Give Example.

A median is the center value of a given list of observations when arranged in an order.
For example, a list of observations is 33, 55, 77, 22, 11.
Arranging in ascending order, we get:
11,22,33,55,77
Hence, the median is 33.

What is the median of two numbers?

If the number of given set of observations is 2, then we have to apply the formula of median for even number of observations, i.e.
Median = [(n/2)th term + {(n/2)+1}th term]/2
Example: Median of 15 and 20 is: [(15)+(20)]/2 = 35/2 = 17.5

What is the median of 10 number of observations?

The median of 10 numbers of observations is: (5th term + 6th term)/2

What is the median of odd numbers of observations?

The formula to find median of odd number of observations is:
Median = [(n+1)th term]/2
Where n is the number of observations.

What is the difference between mean and median?

Median is defined as the center value of ordered list of values.
Mean is the ratio of sum of list of values and number of values, order of values does not matter.

Mode

In statistics, the **mode** is the value that is repeatedly occurring in a given set. We can also say that the value or number in a data set, which has a high frequency or appears more frequently, is called mode or **modal value**. It is one of the three measures of central tendency, apart from mean and median. For example, the mode of the set {3, 7, 8, 8, 9}, is 8. Therefore, for a finite number of observations, we can easily find the mode. A set of values may have one mode or more than one mode or no mode at all.

In this article, you will understand the meaning of mode in statistics, formula for mode for grouped data and how to find the mode for the given data, i.e. for ungrouped and grouped data along with solved examples in detail.

Table of Contents:

Mode Definition in Statistics

A mode is defined as the value that has a higher frequency in a given set of values. It is the value that appears the most number of times.

Example: In the given set of data: 2, 4, 5, 5, 6, 7, the mode of the data set is 5 since it has appeared in the set twice.

Statistics deals with the presentation, collection and analysis of data and information for a particular purpose. We use tables, graphs, pie charts, bar graphs, pictorial representation, etc. After the proper organization of the data, it must be further analyzed to infer helpful information.

For this purpose, frequently in statistics, we tend to represent a set of data by a representative value that roughly defines the entire data collection. This representative value is known as the measure of central tendency. By the name itself, it suggests that it is a value around which the data is centred. These measures of central tendency allow us to create a statistical summary of the vast, organized data. One such measure of central tendency is the mode of data.

Bimodal, Trimodal & Multimodal (More than one mode)

- When there are two modes in a data set, then the set is called **bimodal**

For example, The mode of Set A = {2,2,2,3,4,4,5,5,5} is 2 and 5, because both 2 and 5 is repeated three times in the given set.

- When there are three modes in a data set, then the set is called **trimodal**

For example, the mode of set A = {2,2,2,3,4,4,5,5,5,7,8,8,8} is 2, 5 and 8

- When there are four or more modes in a data set, then the set is called **multimodal**

Also, read:

- Mean Median Mode
- Statistics
- Statistics For Class 10
- Statistics Class 11

Mode Formula in Statistics (Ungrouped Data)

The value occurring most frequently in a set of observations is its mode. In other words, the mode of data is the observation having the highest frequency in a set of data. There is a possibility that more than one observation has the same frequency, i.e. a data set could have more than one mode. In such a case, the set of data is said to be multimodal.

Let us look into an example to get a better insight.

Example: The following table represents the number of wickets taken by a bowler in 10 matches. Find the mode of the given set of data.

Match No.	1	2	3	4	5	6	7	8	9	10
No. of Wickets	2	1	1	3	2	3	2	2	4	1

It can be seen that 2 wickets were taken by the bowler frequently in different matches. Hence, the mode of the given data is 2.

Mode Formula For Grouped Data

In the case of grouped frequency distribution, calculation of mode just by looking into the frequency is not possible. To determine the mode of data in such cases we calculate the modal class. Mode lies inside the modal class. The mode of data is given by the formula:

$$Mode = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

Where,

l = lower limit of the modal class

h = size of the class interval

f_1 = frequency of the modal class

f_0 = frequency of the class preceding the modal class

f_2 = frequency of the class succeeding the modal class

Let us take an example to understand this clearly.

How to Find the Mode

Let us learn here how to find the mode of a given data with the help of examples.

Example 1: Find the mode of the given data set: 3, 3, 6, 9, 15, 15, 15, 27, 27, 37, 48.

Solution: In the following list of numbers,

3, 3, 6, 9, 15, 15, 15, 27, 27, 37, 48

15 is the mode since it is appearing more number of times in the set compared to other numbers.

Example 2: Find the mode of 4, 4, 4, 9, 15, 15, 15, 27, 37, 48 data set.

Solution: Given: 4, 4, 4, 9, 15, 15, 15, 27, 37, 48 is the data set.

As we know, a data set or set of values can have more than one mode if more than one value occurs with equal frequency and number of time compared to the other values in the set.

Hence, here both the number 4 and 15 are modes of the set.

Example 3: Find the mode of 3, 6, 9, 16, 27, 37, 48.

Solution: If no value or number in a data set appears more than once, then the set has no mode.

Hence, for set 3, 6, 9, 16, 27, 37, 48, there is no mode available.

Example 4: **In a class of 30 students marks obtained by students in mathematics out of 50 is tabulated as below. Calculate the mode of data given.**

Marks Obtained	Number of student
10-20	5
20-30	12
30-40	8
40-50	5

Solution:

The maximum class frequency is 12 and the class interval corresponding to this frequency is 20 – 30. Thus, the modal class is 20 – 30.

Lower limit of the modal class (l) = 20

Size of the class interval (h) = 10

Frequency of the modal class (f_1) = 12

Frequency of the class preceding the modal class (f_0) = 5

Frequency of the class succeeding the modal class (f_2)= 8

Substituting these values in the formula we get;

$$Mode = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h = 20 + \left(\frac{12 - 5}{2 \times 12 - 5 - 8}\right) \times 10 = 26.364$$

Mean Median Mode Comparison

Mean	**Median**	**Mode**
Mean is the average value that is equal to the ration of sum of values in a data set and total number of values. Mean = Sum of observations/Number of observations	**Median** is the central value of given set of values when arranged in an order.	**Mode** is the most repetitive value of a given set of values.
For example, if we have set of values = 2,2,3,4,5, then;		
Mean = (2+2+3+4+5)/5 = 3.2	**Median = 3**	**Mode = 2**

Mode Median Mean Formula

There exists an empirical relationship between mode, median and mean and this can be expressed using the formula:

Mode = 3 Median – 2 Mean

Practice Problems

1. Find the mode of the following marks (out of 10) obtained by 20 students:
   4, 6, 5, 9, 3, 2, 7, 7, 6, 5, 4, 9, 10, 10, 3, 4, 7, 6, 9, 9
2. Find the mode for the following data set.
   41, 39, 48, 52, 46, 62, 54, 40, 96, 52, 98, 40, 42, 52, 60
3. Find the mode of the given distribution.

Class Interval	10 – 25	25 – 40	40 – 55	55 – 70	70 – 85	85 – 100
Frequency	12	9	17	16	20	16

To know more about measures of central tendency and mode of data, download BYJU'S – The Learning App.

Frequently Asked Questions – FAQs

What is mode in statistics?

A mode, in statistics, is defined as the value that has higher frequency in a given set of values. It is the value that appears the most number of times.

How to find mode for given set of values?

If we have a set of values equal to 33,44,55,55,66. Then the most repeated value in the given set is 55. Therefore, mode of the given set is 55.

Can there be two modes in a given set of data?

Yes, there can be two modes in a given set of data. Such values are called bimodal.

What is trimodal and multimodal mode?

If there are three modes in a data set, then it is called trimodal and if there are four or more than four modes then it is called multimodal mode.

What is no mode condition?

If the given set of observations do not have any value that is repeated in the set, more than once, then it is said to be no mode.